The Combination of Traditional Teaching Method and Problem Based Learning

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Abstract

In this article, I will discuss some advantages and disadvantages of problem based learning (PBL). Then some differences between mathematics and other fields of science are discussed, using some examples to show the main points in mathematics teaching, and also using examples of PBL in mathematics teaching. Finally, I will discuss some of the skills of mathematics teaching.

There is a trend nowadays to refer to problem based learning simply as PBL and it has been acknowledged as very important in our teaching and in students’ learning. However, I will make a case for combining PBL with conventional teaching for the teaching of mathematics.

In China, there is an ancient great educator named Confucius, who was greatly respected by a lot of Chinese people, especially ancient Chinese people. More than 2000 years ago, Confucius had a lot of things to say about how to teach and how to study. Another Chinese educator Han Yu said, “A teacher is to propagate the doctrines of the ancient sages, give academic instructions, and remove doubts in life”. There are some people who want to abandon almost conventional education. In my own opinion, there must be something good in it since it has lasted for thousands of years. The question is how can we distinguish what is good for teaching and learning now, from what should be adapted or abandoned?

Problem Based Learning

In the article “Overview of problem-based learning” (which can be found at http://www.it.usyd.edu.au/~judy), Associate Professor Judy Kay writes that learning in PBL has the following characteristics:

1. Learning in Context  
   Skills are learnt in order to solve a problem. In this it closely resembles the “real-world” situations in which those skills would normally be required.

2. Problems Motivate the Learning  
   Unlike conventional approaches to education, in which you are told what to learn and then expected to solve a (usually meaningless) problem in order to test your recollection of the information, PBL presents students with authentic problems which motivate their learning as needed to address the issues which the problem generates.

3. Integrated Learning  
   As learning is motivated by the problem it will therefore not be limited to a rigid curriculum.

4. “Ownership” of the Problem  
   There is usually more than one pathway to a goal. PBL acknowledges this in allowing learners to give their own meaning to the problem, and make choices based on this.
Teachers serve to guide and facilitate, but the ownership of the problem is given to the student.

5. Self-Directed Learning
Students will be largely responsible for the learning in which they engage. The problem (plus some additional small tasks) provides essential support; however, students will navigate through the material on their own motivation.

6. Learning about learning
PBL also focus on the learning process itself. As well as learning through problem solving, students will be asked to reflect on the process of learning about problem solving.

7. Collaborative work
PBL takes advantage of the well-known benefits of group-based work to encourage a stimulating environment for learning.

8. Ill-specified problem
Problems are authentic in that they are not artificially constrained to fit into a small design space. That is, there is not necessarily a single “correct” answer to the problem. The problem is left open to interpretation, and is expected to be redefined in accordance with a growing familiarity with it.

9. Recognition of Prior Learning
Students are not blank slates, and enter a course with a variety of skills, experiences, and conceptions.

Some disadvantages of PBL

Academic achievement
Few academics doubt the ability of students schooled in problem based learning to exhibit strong reasoning and team building skills. Concern has been raised, however, over the breadth of content covered. Because the focus of problem based learning centers on a specific problem, academic achievement scores often favor traditional teaching methods when standardized tests are used, but favor neither method when non-standardized forms of assessment are employed (Vernon and Blake 1993). These measures include problem-solving ability, interpersonal skills, peer-faculty relationships, the ability to reason, and self-motivated learning. In contrast, traditional instruction is judged better in the coverage of science content areas (Albanese and Mitchell 1993; Vernon 1995) and in evaluating student’s knowledge content. Although problem based learning tends to reduce initial levels of learning, it improves long-term retention (Farnsworth 1994).

Time demands
Although students generally favor problem based learning courses, and their ability to solve real-life problems appears to increase compared with traditional instruction, instructors have not resoundingly supported the movement toward this type of learning. Contributing to this divergence is the time requirement placed upon faculty to assess student learning (Delafuente, Munyer, Angaran and Doering 1994; Vernon 1995), and to prepare course materials. Also the reduction in coverage of course material necessitated by problem based learning is of concern to some instructors.

Role of the student
An unanticipated problem with problem based learning is the traditional assumptions of the student. Most students have spent their previous years assuming their teacher was the main disseminator of knowledge. Because of this orientation towards the subject-matter expertise
of their instructor and the traditional memorization of facts required of students, many students appear to have lost the ability to “simply wonder about something” (Reithlingshoefer 1992). This is especially seen in first year students who often express difficulties with self directed learning. (Schmidt, Henny and de Vries 1992).

**Role of the teacher**
Instructors in problem based learning curriculum need to alter their traditional teaching methods of lectures, discussions, and asking students to memorize materials for tests. In problem based learning, the instructor acts more as a facilitator than as a disseminator of information. As such, instructors focus their attention on questioning student logic and beliefs, providing hints to correct erroneous student reasoning, providing resources for student research, and keeping students on task. Because this role will be foreign to some teachers, they may have trouble breaking out of their past habits.

**Appropriate problems**
Generating the proper question is the most critical aspect of PBL. Without problems that encompass both a large goal and specific objectives which students must find on their way to reaching the goal's solution, there is a good chance that important information will not be studied. In a study that correlated student directed study and faculty objectives, it was found that students did not stay on track and many important objectives were omitted (Dolmans, Gijselaers and Schmidt 1992). It has even been speculated that if students divert from their anticipated directions during their solution generation, they may completely miss the main content if not redirected by their instructor (Mandin 1995).

**The difference between mathematics and other fields of science**
Mathematics and other sciences have many affinities. Some people think that every field of science should be based on mathematics, since mathematics should be always used as a tool in scientific research. But there are some differences between mathematics and other branches of science. For example:
1. There are many real-world problems from Chemistry, Physics, Geology, Biology, and engineering, but in Mathematics the real-world problems are very few.
2. One of the main purposes of mathematics is to train people to think logically and critically, whereas the main purpose of the other branches of science is to solve the problem using mathematics as a tool.
3. Learning mathematics is somewhat like playing games, because mathematics is abstract, whereas the others are deal with concrete material.
4. Different people think about mathematics differently. Some find it fantastic, and some find it boring.

“Mathematics is gymnastics for the brain.” Due to the above differences, I don’t think we can always use PBL in mathematics teaching. The main purpose of mathematics teaching is not only to let the students solve real problems by using the knowledge they have learnt, but also to think about:
1. Why it is true?
2. What is the main method used in proving it?
3. Why has this result come about?
4. Why does the discoverer (mathematician) think about it in this way?
5. Can it be generalized?
6. Are there any potential properties in it?
In order to solve real-world problems, the students must have enough knowledge. And I think we can’t let the students discover everything by themselves. For example, we can’t let the students discover the Newton-Leibniz Formula for the definite integral, since mathematicians and physicists spent very many years finding this fantastic result before it was finally expressed by Newton and Leibniz. The things we can do are to think about the above six questions in relation to the formula for the definite integral. To show the main idea of this article, I’d like to list them below:

1. Why it is true? See the proof.
2. What is the main method used in proving it? From small to infinitely small.
3. Why has this result come about? It is very difficult to calculate definite integrals using the definition. We should find an easy way.
4. Why does the discoverer (mathematician) think about it in this way? Very complex.
5. Can it be generalized? Can’t be generalized by students themselves.
6. Are there any potential properties in it? A lot.

Here is another example. Think about what results if we sum the first n odd integers. That is, 1 + 3 + 5 + … + (2n-1) = ?

We hope the students may give us the answer by using various methods, including the following three:

1. Equal Distance Sequence: Since the distance between every pair of two neighbor items is 2, we can derive the result by using the formula for the sum of the first n items of an equal distance sequence.

\[ 1 + 3 + 5 + \cdots + (2n-1) = \frac{n}{2} (1 + 2n - 1) = n^2 \]

2. Incomplete induction: Consider the following formulas:

\[ 1 = 1^2; \]
\[ 1 + 3 = 2^2; \]
\[ 1 + 3 + 5 = 3^2; \]
\[ \cdots \]

One can guess that \( 1 + 3 + 5 + \cdots + (2n-1) = n^2 \).

3. The Combination of Algebra and Geometry: Consider the following diagram:
We want to plant some trees in square-shaped land, keeping them equally spaced as shown. On the first day, plant 1 tree. On the second day, plant 3 trees. On the third day, plant 5 trees. And so on. Finally, on the nth day, plant 2n-1 trees. Then we have an area with n rows and n columns of trees. Each row contains n trees. So the number of all trees is \( n^2 \).

A good strategy is to allow the students to communicate the method they have discovered. But I’d like to say something further here. Please notice that we can only “hope” that the students will discover the above methods. But what can we do if no one discovers, say, the third method? I don’t think it is a good strategy to give the students a lot of time to find another method to solve the same problem. My strategy will be to simply tell them that there is wonderful way in mathematics, using the combination of geometry and algebra, and then let the students think about the following two important points listed above: “How does the result comes out?” and “Can it be generalized?” Note that we can use all the methods above to derive

\[
2 + 4 + 6 + \cdots + 2n = n(n+1) = n^2 + n
\]

Using PBL in mathematics teaching

I have mentioned above that there are very few real-world problems in mathematics that can be used as PBL problems. But we can use this method in mathematics teaching. I’d like to give an example to show this strategy.

At the beginning of the teaching of Graph Theory, we can introduce the following problem to increase the students’ interest in learning Graph Theory.

1. Introduce the problem of the Seven Bridges of Konigsberg. This is a real problem, which happened in Konigsberg, which is a little town in Lithuania. It makes students feel that they have an opportunity to solve a real problem by using the knowledge they have learnt. The Seven Bridges of Konigsberg can be shown as the following graph:

   ![Seven Bridges of Konigsberg](image)

   The question is: Can one go for a walk in Konigsberg crossing each bridge exactly once?

2. Give 10 minutes or so let the students try to find a solution by themselves. Note that the problem is very difficult since the people who lived in Konigsberg thought about the problem for many years, so I don’t think we can expect the students to give the answer in about 10 minutes.

3. Give the students a simplified graph as follows:
Let the students think about whether they can solve this problem or not by using this graph.
4. If students can give the answer, then let them discuss it. If not, the teacher can give the answer and show how wonderful mathematics is.
5. Let the students try to solve the following problem:
The questions are:

(i) Which is the shortest way of going through every road entering from gate 1 and exiting from gate 2?
(ii) Which is the shortest way of going through every road entering from the same gate? Let the students discuss whether they can use this method to solve other problems.

Some kinds of skills that can be used in mathematics teaching

It has been reported that students can only concentrate on learning in class for the first 15 minutes or so. In China our classes last for 100 minutes, with a ten-minutes break in the middle. We can’t hope that students will listen to us for the whole time. But we certainly hope the students can understand and gain more knowledge to apply in other scientific disciplines. In order to do this, we can use some skills, or “tricks”, during the lecture. These skills or tricks include the following:
1. The fluctuation of voice. It can help the students concentrate in class.
2. Humour or short story (not overused). Let the students relax for a while, and then grasp their attention suddenly. Overuse of this skill may cause the antipathy of talented students.
3. An appropriate amount of exercises with suitable difficulty. It can help the students better understand what they have learnt.
4. Discussion by choosing a proper question. It may lead to deep understanding.
5. Introduce some short but interesting materials to increase the interest of students in mathematics.
6. Compose or introduce some short poems to help the student remember some things, which are very useful. You can’t expect the students to discover something using their empty brain.
7. Communication after class. This can be used to find out what the students would like us to do in the lecture, what their difficulties are, what they can do in the future using the knowledge they have learnt in class, and so on.

Of course there are many other skills one can use in the class to grasp the students’ attention. One can find out what is suitable for their own teaching, and finally to establish his/her own teaching style. If one loves being a teacher, one certainly can do it.

Please remember that you can’t expect students to be interested in everything they have to learn. That would be extravagant hopes. In China, we have a wonderful expression that means “it is impossible for one to be the top without very hard work”. You can use some tricks to encourage the students to learn some things that they might not necessarily want to learn, but that will be useful to them in the future.

**Conclusion**
1. There are lots of beautiful and useful ideas in the conventional teaching style. We should accept them.
2. There certainly are some disadvantages in the conventional teaching style. We should adapt them, or abandon them. That is, we should “accept critically”
3. PBL is a wonderful way of teaching. We should use it as much as we possibly can.
4. We can’t expect to use PBL all the time and in every aspect of the teaching process, because time is limited and the amount of materials the students should learn is very huge.
5. Learning is not always interesting.
6. The teacher should establish his/her own teaching style, which can most benefit students.
7. The teacher must love to be a teacher.

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*Materials were given to all members in Chinese Bilingual Teaching Group by their English teacher.*