Improving the Qualities of Teaching Calculus – By Using Modern Education Theories and Modern Technology

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Background

More and more students enter our university and most of them need to learn calculus. Every semester, we have about 20 teachers and more than 40 graduate students involved in teaching calculus. Every year, we have more than 1500 students enrolled in different calculus courses, including 150 mathematics students and 20 talented mathematics students. Students from mathematics, computer science and college of management take courses of 'mathematical analysis' and others attend 'advanced mathematics'. All our corresponding courses contain two parts: calculus and analysis. We have several published textbooks written by our teachers (Guangzhong and Yunlong 1993; Yuson, Chonghua, Lu and Wangguo 2001). We are undergoing reform of our education system and the teaching of calculus is changing in such a way that its quality will be improved.

There are some circumstances, however, which make it difficult to teach calculus well. Because of the limited education resources we usually have large classes of students, some of which may have more than 2 or 3 hundred students. We have broad different topics in the syllabus, such as set theory, function, sequences and series, limit, continuity, differential and integral calculus, Fourier series, etc. We also confront a class of students with different levels of skill and knowledge, and with different motivations for learning. Students come from different areas of the country, or from overseas, so they often have problems with life and study as they are undergoing transition from high school learning to university study. First year students usually have 30–35 hours course work per week and some do not spend much time on mathematics.

In our teaching, we want to combine basic training with the cultivation of creative abilities. Our principles of training are 'generalist education and teaching by different categories'. Our notions of training are 'Wide scope, solid foundation, emphasis on abilities and innovation'. The aims of the courses are that students should gain a basic knowledge of the concepts and theories of calculus, understand the idea of analysis, develop skill in corresponding computations and learn to work independently. High-level students are expected to have the ability to analyse problems, apply mathematical tools to establish mathematical models, and use calculus to solve the problems with the help of computers. The course must also prepare students for further study in other courses.

We are endeavouring to achieve our target. We are not satisfied with our current teaching. We have tried to help all the students, but some students failed the examination, some students worked very hard but achieved a low mark in the final examination, and some students passed the examination but forgot what they had learnt soon after.

What should we do to benefit every student of the class? How to improve our teaching? We do our teaching mostly in the traditional way, by lecturing and with tutorials. Some of our teachers use multimedia classrooms. We have done a lot to improve our teaching. We have been trying to simplify our textbook, we give students all the key clues to calculus, tell them the background and application and analyse the problems they have in their studies. We check their exercises and correct them, explaining where and how they went wrong, showing evidence of our point of view. We ask them to discuss their study after class. We have comprehensive problems and exercises for them. The final examination tests students' understanding rather than their ability to memorise.

Modern education theories

I have rather systematically learnt, read and discussed some modern theories of teaching, and I know a lot of strategies for good teaching. Some of these we have

used in our teaching but seldom in courses such as calculus for junior students. I think it is time to make conceptual changes in our teaching process towards more constructive learning, allowing students to be more active in our teaching of calculus.

According to the education theories, good teaching should be helpful for the development of intellectual and imaginative powers, understanding and judgment, problem solving skills and communication skills. It should also promote the ability to see relationships, and provide a broad perspective on students' studying. Good teaching should include an inquiring, analytic and creative approach. Teachers should avoid over-dependence on one way of teaching and learning in formal lectures. The strategy of spoon-feeding in lectures and tutorials fosters in students an over-dependence on information selected and provided for them by their lecturers. It is not good for assessment methods to place too high a premium on memory and recall.

Research has shown that some of the good teaching strategies, which lead to deep learning and understanding, are student-centred and problem based learning. But a good strategy will not always result in good outcomes if we misuse it. Also, we should carefully apply new strategies in our teaching that can be well accepted by our students. Students are masters of learning and we teachers are instructors and helpers of them. We must care about what students think and we must teach differently for different classes. Usually that happens in tutorials because students may ask different questions in different classes. Teaching should be based on our teaching objectives and feedback from students. With a large class, it is always difficult to adopt good strategies for diversified needs of students. In this paper, I will attempt to develop some strategies in teaching calculus.

I also noticed that almost none of the publications about new education methodology that I have read show convincingly improved teaching outcomes though educators have put much effort into good teaching. Learning is a life long process and it is hard to estimate the contribution of a single course for a learner's future success. The teaching quality and the learning quality are difficult to measure because the outcomes are slow to be shown.

Modern technology in teaching mathematics

Modern technology, especially computer aided instruction, is widely used in teaching. Teaching material such as lecture notes, syllabus, exercises, tutorial sheets, solutions, handouts, quiz and examination information and so on are put on a web site. Students can prepare well before lectures and they need not copy what the teacher writes on the blackboard. A number of resources for mathematics can be found by searching the web. If students unfortunately miss the class, they can learn by themselves from these materials. Students and teachers can communicate by email, and students can discuss problems over the Internet with each other, or even with people in other universities or other countries.

Many teachers have begun to use mathematical software (*Mathematica*, *Matlab*, *Maple*, *Mathcad* and so on) in mathematics courses. With overheads replacing the blackboard writings, or the combined use of them, lecturers will have more time to explain mathematics. Processing real life data are fast and easy using software, so students will feel mathematics is useful. The ability to solve seemingly complex problems under the instruction of tutors will make students more confident, and using the pictures and animation will make the course more interesting, so that students will not find the class boring. Plotting pictures on a blackboard is time consuming and difficult, whereas using software is accurate, easy and beautiful, even for 3 dimensional graphs. With the help of computers, students will not worry about numeric computation. By using computers, students are learning better and more effectively with deeper understanding.

There are, however, some problems related to using computers in teaching. Firstly, teachers should have a good knowledge of using computers and technical support is needed from the university, which requires the university to spend more money. Secondly, teachers need to do extra work to prepare materials. Also, some students do not like to use computers, and others may become over-dependent on computers and lack the ability to calculate by hand. Before we begin our teaching with computers, we have to make a good plan.

My strategies to modify the teaching of calculus

My strategies rest upon our teaching experience and education theories. I hope my teaching strategies will be more flexible, and will combine traditional teaching approaches with new methodology. The same topic will have some basic principles but I can apply different strategies to cope with different students.

Change passive learning into active learning

I think this is the most important change in our teaching practice. Passive learning is relatively easy for both lecturers and students, but what we teach is carefully selected and usually involves simplified materials. Students passively involved in class will lack an understanding of the mathematical ideas used to solve problems, and will be confused in their future study and work when they encounter problems that are not in textbooks. We are not only teaching mathematical theorems and formulae but also the mathematical way of thinking needed to analyse and solve problems. We should avoid so-called 'high mark but low ability' phenomena. In order to motivate students to become independent and active learners, our program of teaching strategies needs to include methods and tasks, which are interesting and motivating, and require both team and individual learning tasks. Students will be able to be more actively involved in learning by having the opportunity to discuss, to express their ideas, make short presentations and debate their opinions.

For instance, when we talk about some important concepts such as set, functions and limit, students will be asked to give their own examples, describe their characteristics and discuss them each other. We will give some similar statements and ask students to choose the correct one. They must convince each other if they have different opinions. They may have the wrong answers sometimes but once their wrong answer is corrected, the concept will be impressed on them and students will never make the same mistake again.

Some first year students are very shy and are afraid of making mistakes in front of others. Students who make good suggestions and are active in class will be encouraged (by adding marks). Also every student will have the opportunity to give his or her presentation. Students will be asked to do mini-talks occasionally.

A good technique for time control is needed for this strategy. We should have enough time to finish all the topics that are prerequisites for other courses. Students need to acquire sufficient new knowledge before they have the ability to solve problems and meet the need for future courses. Students' active behaviour in class will be restricted, but will be strongly recommended outside lecture time.

Motivate students to learn calculus well

Research shows that good motivation usually produces good learning outcomes. Some students do not know why they should study mathematics. Some will say, 'I am not good at mathematics,' or 'I am not interested in it,' and 'Mathematics is too abstruse to learn'. Some students suppose mathematics is not important in their discipline. We try to show that mathematics is beautiful and useful, and that learning mathematics is interesting. We can show applications of mathematics in the areas in which students are interested. I will spend more time introducing the background of mathematical concepts and the application of mathematical theories. Students will feel mathematics is easy, especially when we can use mathematical software. We will have many examples from physics, biology and economics. We will use geometric concepts that are intuitive and easy to understand. We will also make use of mathematical software to illustrate and help explain how to deal with corresponding problems. With the help of mathematical software, it is easy to plot curves, surfaces, and various kinds of geometric shapes. Students can see them from different angles and they are impressive.

Problem based learning

Problem based learning often leads to deep level learning outcomes. From the first semester of university study we need to remove the cause of students' surface approach. Problem based learning is an instructional strategy in which students confront contextualized, ill-structured problems and strive to find meaningful solutions. There are many successful examples of students achieving a deep level of understanding when working with problems. Some students may perform very well in problem solving, but we cannot expect students to be Newton or Leibniz and be able to discover the principles of calculus. For the reasons mentioned in the background, we must be careful to construct suitable problems so that they are not too difficult. We will give one or more small problems for every topic and one or two large ones. Students will be encouraged to propose their own problems. Lecturers and tutors will discuss the problems with students and give them some suggestions. Students will be required to interpret problems into well-structured mathematical models, and then work out the solutions under the instruction of teachers. The small problems may involve examples of functions, which reflect relations of data from the real world. Students will work independently or in pairs.

For the large problems, we would need to divide a class into groups of approximately five students. (We need to do some investigation to determine a suitable size.) We will give a list of problems and students will choose one, or even propose their own problems from some area, which interests them. Before starting with the problems, each group should work out a plan with a timetable. Lecturers will give them suitable instruction and students will finally write a group report about their work.

An example of such a problem would be to ask the group to find data from economics or from physics experiments, and use a function to 'best fit' the data. Students will be encouraged to use all kinds of resources, such as libraries and the Internet, to find the data and use mathematical software in computation. In the process, students learn to work together to write a report and one representative will give a talk before other groups.

Another example is the problem of position management: as a manager of a bank, how much cash should be prepared for every workday before working hours?

Other examples may come from insurance or investment planning. Such problems can be well treated by using knowledge of other branches of mathematics, such as operations research.

We would not expect students to solve the problem thoroughly. We need students to use calculus to deal with the problem and to write out their schedule of problem solving and to be stimulated in the desire for further study of mathematics.

Advanced topics

For advanced students, we will have very challenging topics such as the proof of the elementary theory of algebra. In my opinion, we spend too much time on the proofs of theorems, since many students are afraid of the proofs, and the teaching is not effective. The problem is that teaching mathematical theorems without proofs often leads to surface level learning. Students only know 'what' and they do not care about 'how' and 'why'. We can simplify our lectures by omitting the proofs that are difficult for normal students. We should only talk about the mathematical ideas of the difficult proofs, and leave rigorous proofs to be written by some advanced student group. Advanced topics will not be included in the final examination.

For example, when we talk about the Taylor expansion of functions we would let students apply it with the function

e^x. According to the theory, the series is convergent for every real number x. But when using this algorithm with some x, the result can be negative (an obviously wrong answer). We must think about the theories of numeric computation. Computers are powerful, fast and accurate, but will make the wrong conclusion if we choose a bad algorithm.

Another example we have already used in our teaching for talented mathematics students is to introduce the Lebesgue integral (Guangzhong and Yunlong 1993) in a new way that avoids tedious measure theory. It is easy to understand and does solve some problems that are hard to solve by mathematical analysis. We can exchange the order of limit, derivative and integral with much weaker conditions.

Discussion on web

We can establish a discussion area like *WebCT* on our Net. Students can put their mathematical problems of interest on it, can ask for help, and can check any results related to learning calculus. Lecturers and tutors can also dispose their opinions. We will ask our university to support our plan. At some fixed time, there will be teachers on the Net to join the discussion and help students, and to get feedback from students. That will be good for both teaching and learning.

Assessment

I will explain the assessment of the course in my first lecture and ask for students' opinions and suggestions. My principles of assessment will mainly be to stimulate students to work hard and produce deep level learning. The final examination will not rely on memory but on understanding. I will introduce some theories of good learning and let students discuss the assessment problem. Every group should write a report and some representatives will be asked to make a presentation of the report. Surface level learning will not get a high mark. Students can choose a surface learning strategy if they just want to pass the examination and do not like hard work.

Because we have a large number of students and our course covers a broad topic, we will mainly assess students' learning outcomes by a final examination in the form of closed book, although we will take account of assignments and performance in the class. Apart from some elementary formulae and important theorems there will be no need to memorise much in the textbook. Understanding mathematical concepts and the ability to solve problems will be very important to pass the examination. The problems in the examination may appear differently from those in the exercises.

One kind of problem might ask students to correct mistakes from several mathematical statements. Students are required to make a judgment of some statements and to write an analysis of their relationships. We could change a mathematical statement slightly and ask students to say whether the mathematical meaning has been changed, and explain why. For example, we could give two statements about a real function f(x) defined on an interval I. One statement would be for f(x) continuous on I, the other for f(x) uniformly continuous. Students would need to judge the relationship of two statements and explain why. Even if we have not talked about uniformly continuous, students will be able to give the correct answer if they have understood the concept of continuous function.

Summary

Reform of teaching is very hard work but it is worth doing because it is both interesting and challenging. In this paper, I have mainly discussed some possible reform in strategies of teaching calculus in Fudan University. Some of them I have used in my previous teaching and some are easy to apply in my opinion. Some strategies may be difficult to implement.

I need to get more ideas of good teaching from other universities such as The University of Sydney. A great deal of care is needed in the implementation and design of my strategies for teaching calculus. I will try these in my future teaching and check the outcomes. I hope I can make progress step by step. I am also willing to share the experiences of teaching calculus with others.

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