

Using pictures to teach linear algebra

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When we teach a mathematics class we must teach the concept, theory and its application. Of these three parts, teaching the concept is the most important part. Teaching theory is the most complex part and the teaching of the application is the most interesting part to the student. But the concept of mathematics often has the property of being abstract and it is difficult to understand for the students. The deduction of mathematics theory is so complicated that the student may well lose interest or patience with the process. How to overcome these difficult problems in our teaching? It is a challenge for every teacher. Here I will emphasize the application of pictures in teaching as a possible approach.

The teaching of mathematics concept

Every concept in mathematics comes from the abstract world. If we want to teach the concept well we must understand the origin of the concept. In order to attract students we should use pictures to illustrate these problems.

In linear algebra the concept **matrix** is often too abstract to understand. Some students cannot understand it even after they successfully pass their examinations. I will use pictures to teach this concept in my class. The first is the production of three companies in four seasons.

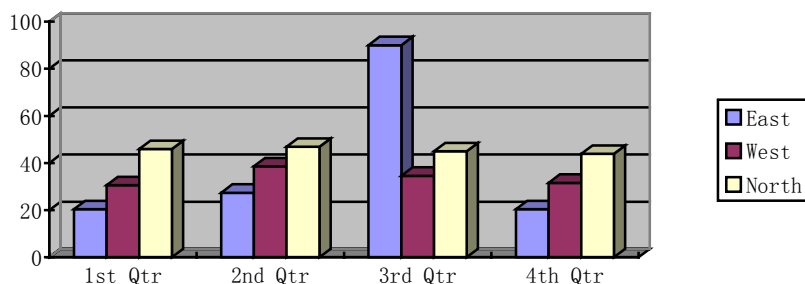


Table 1. Production of three companies in four seasons

The second example comes from the bookshelf of a bookshop and the third example is the temperature of the earth. In these examples we ask the questions ‘In these three examples, what is an efficient method to describe these three phenomenon?’.

1. For the table of production they have two key properties: column and row.
2. If we put all the books on the bookshelf, each book is located in the fixed columns and rows?
3. The temperature of the world according to the area it occupies connects with longitude and latitude on the earth. If we treat longitude and latitude as column and row, the problem of the temperature problem can be recorded correctly.

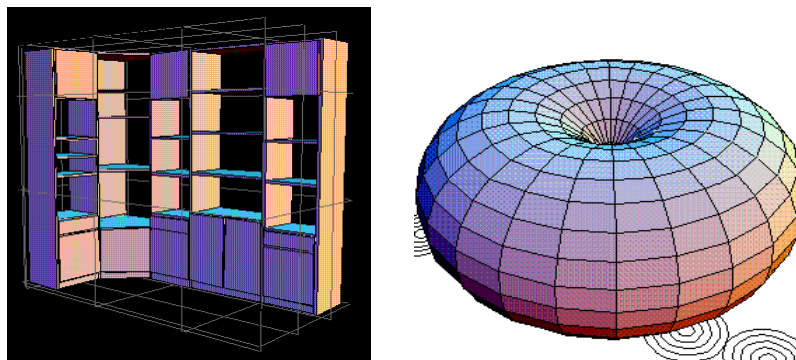


Figure 1. Books on the bookshelf and the temperature on the earth

In these three examples we can abstract their common properties. We can solve them by a table and put the grid in different meaning. Table 1 can be simplified as the following Table 2.

	1st Qtr	2nd Qtr	3rd Qtr	4th Qtr
East	20.4	27.4	90	20.4
West	30.6	38.6	34.6	31.6
North	45.9	46.9	45	43.9

Table 2. Production of three companies in four seasons

In mathematics we need to make the concept abstract and omit the real meaning. For example, we consider these problems in the matrix which contains columns and rows.

$$\begin{pmatrix} 20.4 & 27.4 & 90.0 & 20.4 \\ 30.6 & 38.6 & 34.6 & 31.6 \\ 45.9 & 46.9 & 45.0 & 43.9 \end{pmatrix}$$

Clearly this is a 3×4 matrix and the other two examples can also be described using the same method. That is, they can be expressed with the use of a matrix. With the help of pictures we can introduce the concept of a 'matrix' easily.

The matrix is a table that contains columns and rows and every position in the table contains a number.

In order to understand this matrix concept deeply we can use other pictures and give students a simple application of a matrix. Figure 2 is a picture of an ancient painting and a sculpture of Buddha. These ancient works of art are expensive and their preservation is a big problem for us. Obviously, the protection of the antiquity is not our task. Our problem is how to preserve this expensive antiquity by mathematical methods. We will use matrix theory to record them correctly. First we consider the problem with the painting.

In order to calculate this painting for preservation we separate the painting with different horizontal lines and vertical lines. We will divide the painting into $m \times n$

grids. If we record the colour of every grid position we will record the painting accurately. The more lines, the more accurate the record will be. That is, we can record this expensive painting in a large matrix. A question remains: how to record the sculpture of Buddha into matrices?



Figure 2. Left: A gouache painting in the Cabinet des Arts Graphiques. Right: Sculpture of Buddha.

When we get the matrix concept we must teach the operation and properties of the matrix. In these parts we still use pictures to help our teaching and make students more interested in these parts.

The teaching of mathematics theory

The teaching of mathematics theory is difficult as this part is too complicated to make a reduction. In the teaching of matrix theory, the operation of the matrix, and property of the matrix, we can use the following picture (Figure 3) and connect it with matrix operation. Suppose, some books have been packed in a bookshelf and we move them to another bookshelf. How can we use matrix operation to describe this change? In order to solve this problem we must learn the operation of the matrix.

After we finish the plus, minus and product of the two matrices. We can use matrix operation to describe the transformation of the books.

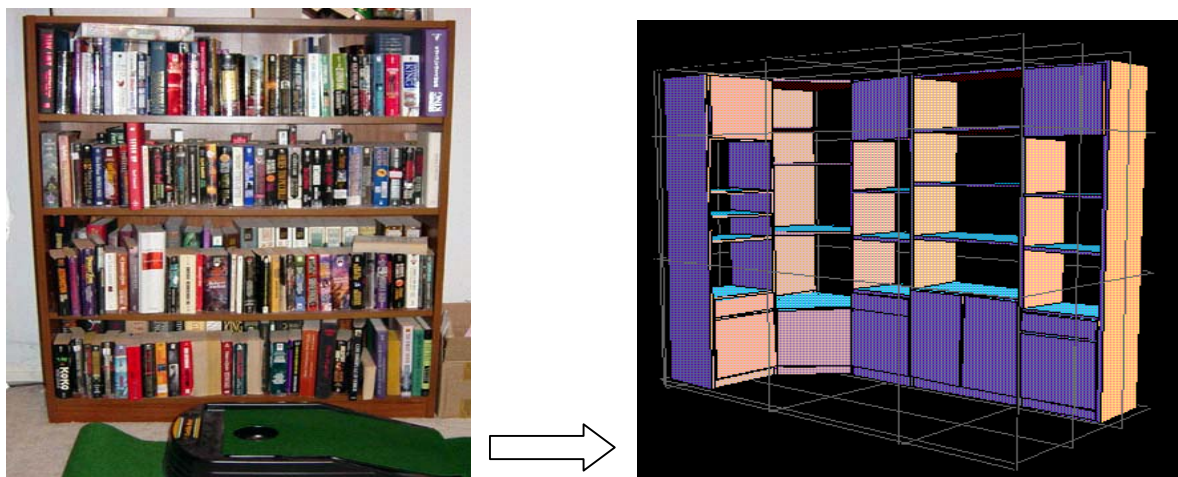


Figure 3. Change the books from one bookshelf to another

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1q} \\ a_{21} & a_{22} & \dots & a_{2q} \\ \dots & \dots & \dots & \dots \\ a_{p1} & a_{p2} & \dots & a_{pq} \end{pmatrix} \Rightarrow \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix}$$

In this process we can use a matrix A to describe the first bookshelf and matrix B to describe the second bookshelf. The transformation between matrix A and B can describe the books changing from one bookshelf to another. That is, the matrix operations have their real meaning. If the

students have become interested in this, we can put the following beautiful mathematics equation as an example of the deduction of the matrix theory: a square root of a unit matrix.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^{\frac{1}{2}} = \begin{pmatrix} g & b & d & e & a & e & d & b & g \\ b & e & e & b & 0 & -b & -e & -e & -b \\ d & e & g & -b & -a & -b & g & e & d \\ e & b & -b & -e & 0 & e & b & -b & -e \\ a & 0 & -a & 0 & a & 0 & -a & 0 & a \\ e & -b & -b & e & 0 & -e & b & b & -e \\ d & -e & g & b & -a & b & g & -e & d \\ b & -e & e & -b & 0 & b & -e & e & -b \\ g & b & d & -e & a & -e & d & -b & g \end{pmatrix}$$

Here the parameters are:

$$a = \frac{1}{\sqrt{5}}; b = \frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{10}}; g = \frac{-1+\sqrt{5}}{4\sqrt{5}}; d = \frac{1+\sqrt{5}}{4\sqrt{5}}; e = \frac{1}{2} \sqrt{\frac{5-\sqrt{5}}{10}}$$

Conclusion

In the above we can see if we use pictures in mathematics teaching we can change the abstract pictures concept into intuitive expression and dull mathematics theory may become far more interesting. In this linear algebra class we not only teach the concept and theory but also the application of the theory. It connects the abstract theory with the real world. In fact many other mathematics teaching classes can use a

picture methodology. It will make mathematics classes more interesting.

Appendix

These beautiful fractal images can be used in teaching and make students more interested in mathematics. The detailed simulation can be seen on the web site. <http://sprott.physics.wisc.edu/fractals.htm>

