

# Astrophysics Option

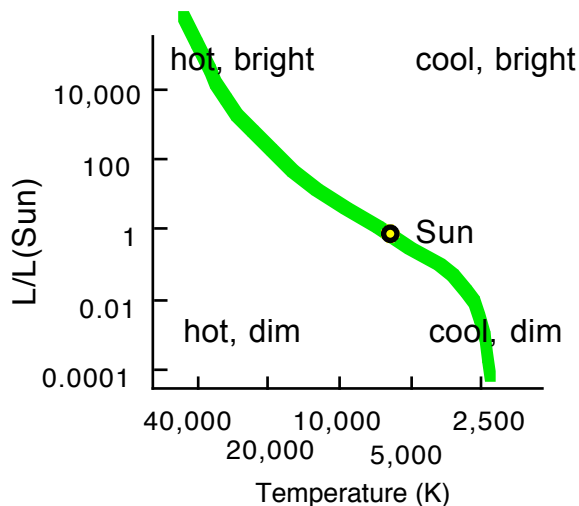
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I want to give you a feeling for how modern astrophysics hangs together. We'll do this by looking at the central unifying diagram in the study of stars: the **Hertzsprung-Russell diagram**. We will see how the H-R diagram is based on observations, using many of the techniques you have studied, and leads to profound understanding about the nature of stars and the central role of gravity in the Universe.

## The observed properties of stars

The H-R diagram was discovered as an *observed* relationship between the properties of stars. Discovered independently by Ejnar Hertzsprung and Henry Norris Russell, it is simply a graph of the luminosity of stars plotted against their temperature.

In its simplest form, an H-R diagram looks like this. The luminosity of stars is plotted on the y-axis, and the temperature on the x-axis. Because of the way the diagram was originally constructed, plotting absolute magnitude against colour, the temperature goes the wrong way, increasing to the left. For historical consistency, this has never been changed.

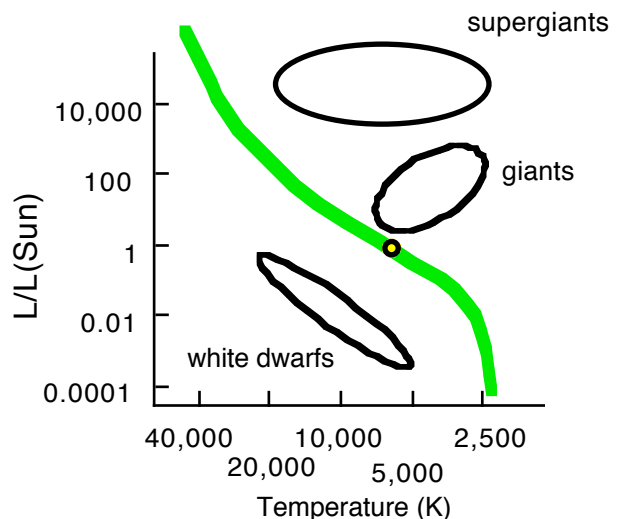


When the luminosities and temperatures of stars near the Sun are plotted on this diagram, 90% of them are found to lie in a narrow band running roughly diagonally across the diagram, from hot and bright (top left) to

cool and dim (bottom right). This band is called the *main sequence*.

Several other distinct groupings of stars can be identified. Most of the remaining stars lie in a band from faint(ish) and moderate temperatures, to extremely bright and cool. To be both cool and very bright, these stars must be enormous: the *giants*. Even brighter than the giants, and hence even larger, are the *supergiants*, which span a wide range of temperatures. Then there are some stars below the main sequence,

white dwarfs.



which are very hot and very faint, which must mean they are very small – the *white dwarfs*.

### Interlude 1: The luminosity of a star

Now, in order to place a star on the H-R diagram, we need to know how bright it is. In particular, we need to distinguish between stars which are intrinsically bright, and stars which appear to be bright because they are close to us.

To understand why this is important, consider two lists of stars: the ten brightest stars in the sky, and the ten nearest stars in the sky.

The <b>brightest</b> stars			The <b>nearest</b> stars		
star	apparent magnitude	d (pc)	star	apparent magnitude	d (pc)
Sirius	-1.50	2.6	Proxima Centauri	11.5	1.3
Canopus	-0.73	96	Alpha Centauri	0.1	1.3
Alpha Centauri	+0.10	1.3	Barnard's Star	9.5	1.8
Vega	+0.04	7.9	Wolf 359	13.5	2.3
Arcturus	0.00	11.6	Lalande 21185	7.5	2.5
Capella	+0.05	13.1	Sirius	-1.5	2.6
Rigel	+0.08	184	Luyten 726-8	12.5	2.7
Procyon	+0.34	3.5	Ross 154	10.6	2.9
Betelgeuse	+0.41	131	Ross 248	12.2	3.2
Achernar	+0.47	45	Epsilon Eridani	3.7	3.3

There is scarcely any overlap between the two! Some bright stars are at enormous distances, and some near stars are very faint.

So the (apparent) brightness of a star is not a very useful quantity. We need something better to describe what the star is like.

The inverse square law means that once we know the distance to a star, we know how bright it really is.

$$M = m - 5 \log(d) + 5$$

where  $d$  is the distance to the star in parsecs.

The *absolute magnitude* of a star is the magnitude it would have if it were at a distance of 10 pc.

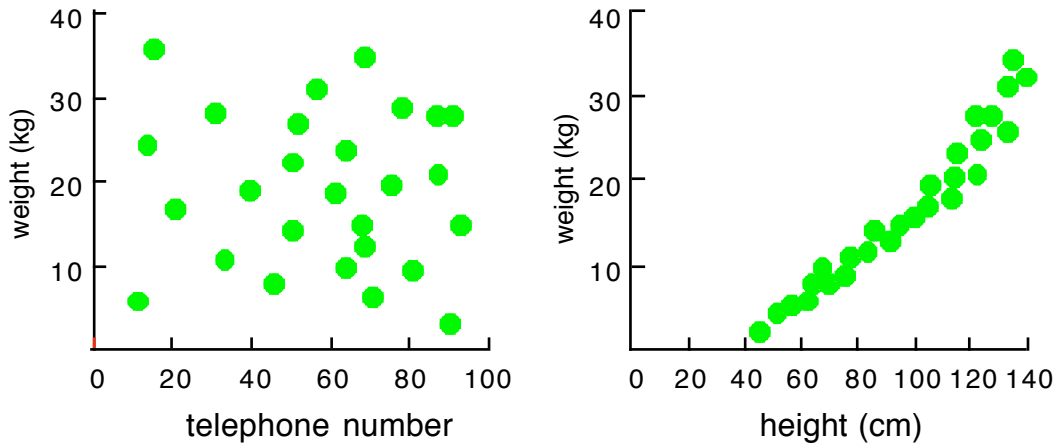
Luminosity and absolute magnitude are equivalent, except that magnitudes are on a logarithmic scale. A difference of 5 magnitudes represents a difference of a factor of 100 in luminosity, so a difference of 1 magnitude represents a factor of  $100^{1/5} = 2.512$  in luminosity.

## The relation between brightness and temperature

Stars do not fall randomly on the H-R diagram: they are apparently only allowed to have *some* values of temperature and luminosity.

This tells us there is an underlying physical link between these properties.

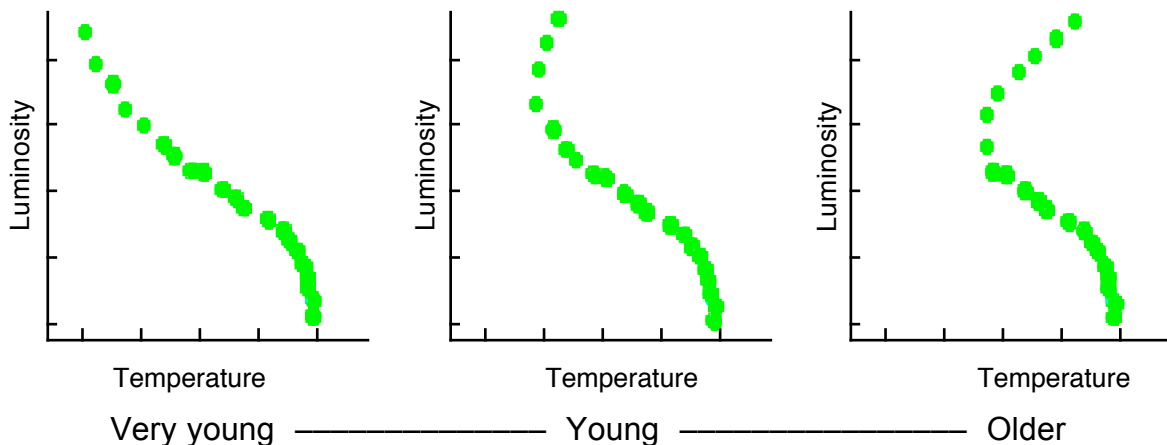
For example, consider an alien scientist constructing a similar diagram for people. Imagine zyx took a group of school children (being an easily studied population) and plotted their weight against every numerical factor zyx could find.



The left diagram shows the plot of weight vs. the last two digits of their telephone number. Zyx finds no correlation between them (which doesn't surprise us earthlings terribly!). However, when zyx plots the children's weight against their height, zyx finds a strong correlation between the two. This strongly suggests there is a single underlying factor which makes the weight of a child correlate with his or her height.

A little further investigation on the part of our intrepid scientist would lead zyx to conclude that the underlying factor is the *age* of the child. Zyx discovers this by noting that kindergarten children almost all have small heights and weights, while 4th graders have, by and large, larger heights and weights. What's more, a long observation shows that a single child traces the curve as she or he grows.

So is this what is happening for stars? Does the main sequence represent an *age*

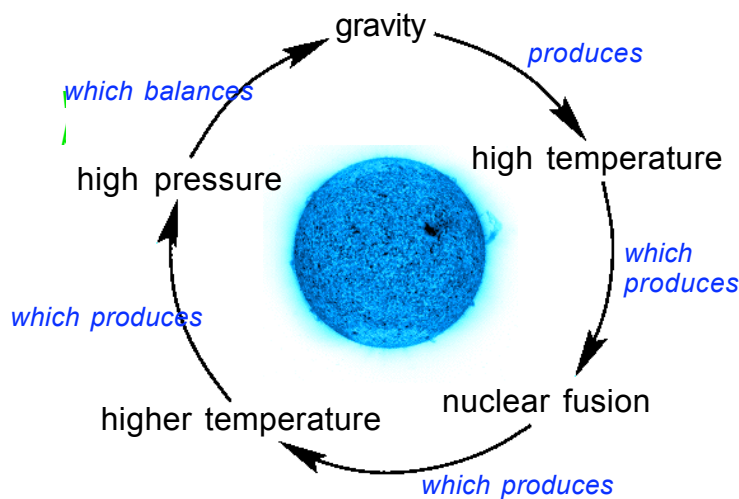


sequence, where a single star starts out (say) hot and bright, and gradually cools and dims as it ages (or vice versa)?

**No.** We can see the main sequence is not an age sequence by looking at groups of stars which we know to be all the same age: stars in clusters.

Clearly stars of the same age are not all appearing in the same place in the diagram. So whatever the underlying factor is, it is not age.

In fact, it is **mass**. The main sequence is a mass sequence, not a time sequence. All the properties of a star – its temperature, luminosity, radius etc. – are dictated by its *mass*.



### Why mass?

Why is mass the important factor for stars?

Because the whole of a star's life is a sequence of attempts to hold off *gravity*. Gravity compresses the matter in the centre of the star until it is hot enough to fuse hydrogen into helium. This produces enough energy and hence pressure to counteract gravity: the star is in *equilibrium*.

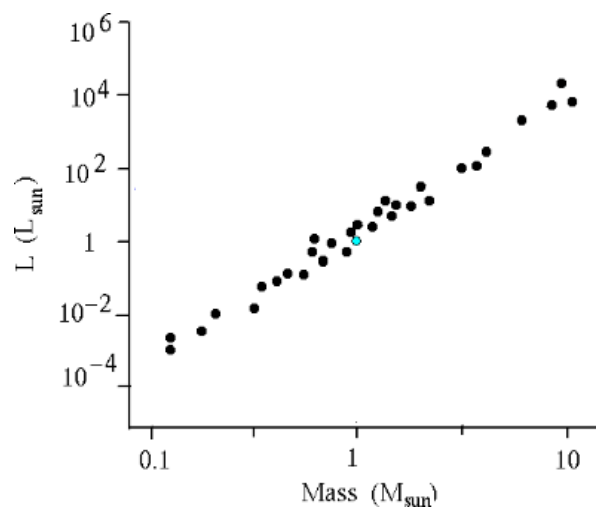
Fusion requires a temperature of 10 million K to start. Above this, a small increase in temperature results in a *large* increase in the rate of fusion.

So why are *heavier* stars *brighter*?

Consider increasing a star's mass by pouring some extra hydrogen on.

- More mass → more compression
- higher temperature
- *much* higher rate of fusion
- *much* higher luminosity

A small increase in mass leads to a large change in the luminosity.



If we plot a star's mass against its luminosity, we find a relation

$$L \propto M^{3.5}$$

Which means that if we increase the mass of the star by a factor of 10, we increase its luminosity by a factor of 3000.

So massive stars are a *lot* brighter than their lower-mass siblings.

## Interlude 2: Measuring the mass of a star

How do we measure the mass of a star?

We need to use stars in binaries. If we know the period and separation of the binary, we can use Newton's laws to derive the masses of the stars:

$$m_1 + m_2 = \frac{4\pi^2 r^3}{GT^2}$$

There are three types of binaries:

- *Visual binaries*: very wide binaries where we can actually see the two stars moving around one another.
- *Spectroscopic binaries*: where we see the Doppler shifts of the two stars as they move around each other.
- *Eclipsing binaries*: where we see the light from the stars change as one star moves in front of the other.

## Live fast, die young

How long a star can burn hydrogen on the main sequence depends on two things:

- how much hydrogen it has; and
- how fast it burns it

The first is just the star's mass, and the second the star's luminosity.

Thus the lifetime of a star goes like

$$t \propto \frac{M}{L} \propto \frac{M}{M^{3.5}} = M^{-2.5}$$

In other words, a factor of 10 increase in mass corresponds to a *decrease* in the lifetime of the star by a factor of 300. So massive stars live for much shorter times than less-massive stars. This explains the shape of the HR diagrams for clusters: the more massive stars leave the main sequence earlier, so the main sequence gets progressively eroded, like a candle burning down.

## After the main sequence: the beginning of the end

What happens when the star runs out of hydrogen in its core?

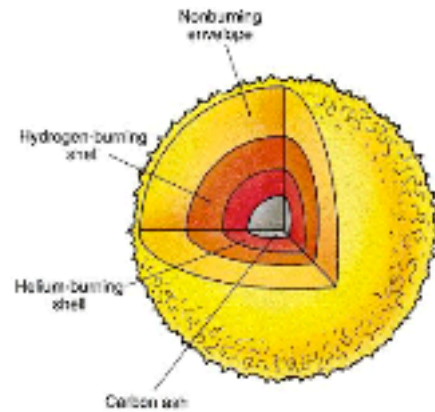
Hydrogen fusion stops, so the star is no longer producing the energy which counteracts gravity.

Gravity takes over again, and the core starts to collapse.

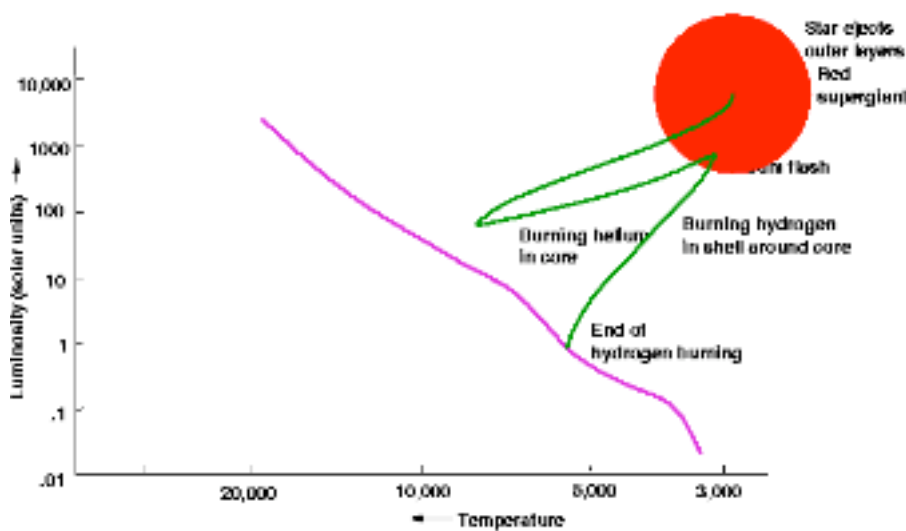
The core heats up, and two things happen. Unburnt hydrogen around the core ignites for the first time, causing the outer layers of the star to puff up enormously.

The star becomes enormously large and red: a *red giant*. The core continues to contract, until the temperature becomes hot enough to ignite helium. This halts the collapse for a while, as the star burns helium in its core, just as it burnt hydrogen on the main sequence.

And here we see how we can use the Hertzsprung-Russell diagram as a tool for understanding the internal evolution of stars: we can understand how a star, during its life, moves to different regions of the diagram in response to changes in its internal structure.



The cycle then repeats itself. Helium is fused to carbon and oxygen in the star's core, but eventually the core runs out of helium. Fusion stops, and the core



resumes its collapse. This heats up the layer of helium just outside the core, which begins fusing in a shell of its own, inside the hydrogen burning shell. This extra energy forces the envelope to expand again, and the star begins a second ascent of the giant branch:

the *asymptotic giant branch*.

We can predict what happens next: the cycle repeats again and again, each time starting to fuse heavier and heavier elements to stave off the next collapse. The core reaches high temperatures by converting gravitational energy into thermal energy.

Each time the star runs out of fuel, the inexorable collapse due to gravity begins again.

### The end of the line

Here, our story splits into two different endings.

- *Low mass stars* (like our Sun) never get hot enough to ignite carbon in their cores. So once the hydrogen is all burnt, the outer layers are ejected and the core collapses to form a *white dwarf*.

The ejected outer layers are seen as a planetary nebula, an expanding shell

of gas lit by the white dwarf within.

As the outer layers of the star are removed to expose the hot core, the star becomes much bluer. Once the core, now a *white dwarf*, has shrunk to its final size, it just cools down.

A white dwarf maintains itself against the gravitational pull because of *degeneracy pressure*: the electrons are squeezed so tightly together that they behave like a solid, and this is stiff enough to resist the force of gravity.

- *High mass stars* (greater than about 6 solar masses) continue to burn all elements up to iron. These elements collect and burn in concentric shells, like an onion. During this time, the star zigzags across the H-R diagram.

After iron, however, there is no more energy to extract from fusion. Once the silicon in the core has fused to make iron, the star can no longer support itself against collapse. The iron core, now weighing about 1.4 times the mass of the Sun, collapses inwards again; but this time there is no new fusion process to stop the collapse. The end is sudden and violent: a supernova explosion.

The energy released in the supernova explosion is so vast that the supernova can outshine a whole galaxy.

This time, electron degeneracy pressure cannot resist the gravitational pressure of the outer layers. The electrons are forced to combine with protons to form neutrons. The neutrons are squeezed so tightly together that *neutron degeneracy* sets in, and this time this is stiff enough to resist the gravitational pressure. Now the force of gravity is being resisted by the strong nuclear force: in essence, the whole core is like one giant atomic nucleus! A *neutron star* has been born.

### Peace at last: stable states

Like a white dwarf, a neutron star is a truly stable object: it can exist forever, without the need for fuel to hold itself up. This is the third stable state of matter:

- In **planets** (and ordinary things around us), gravity is resisted by intact electron shells of ordinary atoms (normal matter).

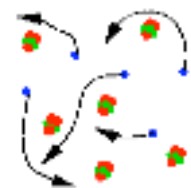
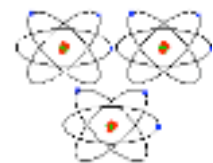
Mean density: 10 g/cc

- In **white dwarfs**, electrons are forced so close together that they become rigid; their electromagnetic repulsion resists gravity (electron degeneracy).

Mean density:  $10^8$  g/cc

- In **neutron stars**, neutrons are forced into virtual contact, and the strong nuclear force resists gravity (neutron degeneracy).

Mean density:  $10^{14}$  g/cc



Just as white dwarfs have a maximum allowed mass, so do neutron stars. There comes a mass when even the outward pressure provided by degenerate neutrons cannot counterbalance the inward pull of gravity.

And after that, there is no force left which can balance it. Gravity has won: the star must collapse to a *black hole*.

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So that's the story of the life of stars. I hope you have seen how the study of the observed properties of stars leads to profound insight into the nature of stars, how they live and die, and the over-arching importance of gravity in this whole cycle.