Blackbody Radiation

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1. Introduction

Blackbody radiation is radiation in thermal equilibrium with the walls of an enclosure kept at a fixed temperature. Blackbody radiation can be thought of as a gas of photons, and correspondingly its properties require a quantum mechanical description. In this lecture the history of the understanding of blackbody radiation is briefly discussed, and a derivation of the blackbody (Planck) spectrum is given. Kirchoff's law and several other laws relating to the spectrum are also derived, and the role of the Planck spectrum in astrophysics is discussed, with emphasis on the basic physics.

2. Historical background

In 1859 Gustav Kirchoff proved a theorem based on thermodynamics which posed a challenge to theoretical and experimental physicists. Kirchoff considered a body in thermal equilibrium with radiation such that the energy the body absorbs is converted into thermal form only. Let E_ν denote the amount of energy emitted by the body per unit time, per unit frequency ν, and per unit area. If A_ν denotes the absorption coefficient, i.e. the fraction of incident energy absorbed by the body, then Kirchoff's theorem states that the ratio E_ν/A_ν depends only on ν and the temperature:

$$\frac{E_\nu}{A_\nu} = B_\nu(T). \tag{1}$$

Kirchoff defined a *blackbody* as one for which $A_\nu = 1$. He also gave a working definition of perfect blackbody: radiation inside an enclosure with walls at a fixed temperature "is constituted, with respect to quality and intensity, as if it came from a perfect blackbody." If a small hole is drilled in the wall of the cavity, any radiation falling into the hole is unlikely to emerge, since it will be reflected and eventually absorbed by the walls of the cavity. The radiation emerging from the hole is in thermal equilibrium with the walls, and so it follows that the radiation from the hole is that of a perfect blackbody.

Kirchoff's challenge to physicists was to determine the function $B_\nu(T)$, the blackbody spectrum. Experimentally this was a difficult task, and the complete form of the function did not become apparent until the late 19th century. Prior to that, certain characteristics of the blackbody spectrum were determined. In 1879 Josef Stefan conjectured on empirical grounds that the total power radiated by a hot body varies as the fourth power of temperature. In fact this is strictly true only for a blackbody, and in 1884 Boltzmann proved the Stefan-Boltzmann law, using thermodynamics and the electromagnetic theory of Maxwell. Around the same time it was established theoretically that

$B_\nu(T)$ has a single maximum, which moves to lower frequencies with decreasing temperature (the Wien displacement law).

On the theoretical side, attempts to derive $B_\nu(T)$ based on classical electrodynamics and thermodynamics gave a result which did not make any sense, and disagreed with observations.

In 1900, Max Planck used new measurements at short wavelengths that had just become available to 'guess' the functional form for the spectrum

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_BT}-1}, \tag{2}$$

where $h \approx 6.63\times10^{-34}\,\mathrm{J\,s}$ is the Planck constant and $k_B \approx 1.38\times10^{-38}\,\mathrm{J\,K^{-1}}$ is Boltzmann's constant. Equation (2) — the Planck spectrum — fitted all available measurements, and was an inspired guess! Planck also gave a derivation for this expression, which involved a number of steps which could not be justified in the classical scheme. Atoms were assumed to be oscillators that absorb or emit discrete amounts of energy proportional to $h\nu$. Also, the parcels of energy were counted as indistinguishable objects. With the benefit of hindsight we recognise the parcels of energy as photons, and the method of counting as an account of the statistics of photons. However, Planck's derivation was not completely correct (the 'photons' could have energies $mh\nu$, where m is an integer) and the correct quantum mechanical derivation was later given by Bose and Einstein.

In the next section a modern derivation of the Planck spectrum is given, based on the description of a Planck radiation field as a gas of photons. We will derive the energy density $\rho_\nu(T)$ in a blackbody radiation field, which is related to $B_\nu(T)$ as follows. The quantity $B_\nu(T)$ may be defined as the energy passing a given point in a blackbody radiation field per unit time, per unit area, per unit frequency, and per unit solid angle, in a given direction $\boldsymbol{\kappa}$ (formally a quantity with these units is an *intensity*). The definition involves 'per unit solid angle,' which may require explanation. In considering the transport of energy by radiation in a direction $\boldsymbol{\kappa}$ it is necessary to consider a bundle of rays within a small range of angles $d\Omega = \sin\theta\, d\theta\, d\phi$ about $\boldsymbol{\kappa}$, where θ and ϕ are the usual spherical polar angles about the axis defined by κ. The quantity $d\Omega$ is called an element of solid angle. Since $B_\nu(T)$ is isotropic, it is independent of the choice of direction $\boldsymbol{\kappa}$. To obtain the energy density, note that in a time dt the amount of energy (per unit frequency and per unit solid angle) crossing an element of area dA perpendicular to $\boldsymbol{\kappa}$ is $B_\nu(T)dA\,dt$. All of this energy is inside a cylinder with cross sectional area dA and length $c\,dt$, so the energy density associated with radiation in the direction $\boldsymbol{\kappa}$ is $B_\nu(T)\,dA\,dt/(dA\,cdt) = B_\nu(T)/c$. Finally the total energy density is obtained by integrating over all solid angles, and since $\int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi = 4\pi$ we obtain

$$\rho_\nu(T) = \frac{4\pi}{c}B_\nu(T). \tag{3}$$

3. Origin of the Planck spectrum

3.1 The Boltzmann law

The derivation of the Planck spectrum relies on the Boltzmann law, a result from statistical mechanics. This may be justified, at least heuristically, by appeal to a simple system in thermodynamic equilibrium: a gravitationally stratified atmosphere at a constant temperature T (Feynman, Leighton & Sands, 1963). For such an atmosphere the gravitational force per unit volume is balanced by the pressure gradient,

$$\frac{dP}{dz} = -\rho g \tag{4}$$

where $P = nk_BT$ is the pressure, n being the number density, z is the coordinate in the vertical direction, $\rho = nm$ is the mass density, where m is the mass of the molecules comprising the atmosphere, and g is the acceleration due to gravity. The solution to this differential equation is

$$n = n_0 e^{-mgz/k_BT}. \tag{5}$$

Feynman points out that n/n_0 is proportional to the probability of a molecule being at a height z, and mgz is the potential energy of a molecule at height z. Hence the probability of a molecule having a certain potential energy is proportional to

$$\exp[-(\text{potential energy of molecule})/k_BT].$$

Feynman demonstrates that this result does not depend on the specific form of the force law, i.e. it would hold for any conservative force.

The preceding may be seen as a plausibility argument for the classical form of Boltzmann's law: in thermodynamic equilibrium the probability of finding molecules in a given spatial arrangement varies as the negative exponential of the potential energy of the configuration divided by k_BT.

The result for quantum mechanics is essentially the same. A system bound by a potential will have a discrete set of energy levels $E_0, E_1, E_2, ...$ The probability of the system being in a raised state E_i relative to the probability of being in the ground state E_0 is proportional to $e^{-(E_i-E_0)/k_BT}$.

Both forms of Boltzmann's law may be derived from thermodynamics based on the definition of entropy. However, the derivation is beyond the scope of the present discussion.

3.2 Derivation, based on the Boltzmann law

Consider a photon gas in thermal equilibrium inside an enclosure. We start by considering a hypothetical system consisting of photons at a fixed frequency ν. For this system the number of photons in the enclosure is variable; the photons are continuously being emitted and absorbed by the walls. If there are

n photons, the system is in a state with energy $E_n = nh\nu$. According to the Boltzmann law, the probability of this state is

$$p_n = \frac{e^{-\beta E_n}}{\sum_{i=0}^{\infty} e^{-\beta E_i}}, \tag{6}$$

where we have explicitly identified the normalization factor to ensure $\sum_{n=0}^{\infty} p_n = 1$, and where the notation $\beta \equiv 1/k_B T$ is used.

The average energy $\overline{E}$ of the system is given by

$$\overline{E} = \sum_{n=0}^{\infty} p_n E_n = \frac{\sum_{n=0}^{\infty} E_n e^{-\beta E_n}}{\sum_{i=0}^{\infty} e^{-\beta E_i}} = -\frac{\partial}{\partial \beta} \ln \sum_{n=0}^{\infty} e^{-\beta E_n}, \tag{7}$$

where the last step is a clever trick. The sum in the last line is a geometric series and may be evaluated to give

$$\sum_{n=0}^{\infty} e^{-\beta E_n} = \frac{1}{1 - e^{-\beta h\nu}}. \tag{8}$$

Hence Equation (7) leads to

$$\overline{E} = \overline{n} h\nu, \quad \text{where} \quad \overline{n} = \frac{1}{e^{\beta h\nu} - 1}. \tag{9}$$

We can identify $\overline{n}$ as the average number of photons in the enclosure for our single-frequency system.

So far we have concentrated on photons of a given frequency, but of course there is a continuous range of frequencies of photons. If D_ν denotes the number of photon states per unit frequency, then the energy density in an enclosure with volume V is

$$\rho_\nu(T) = \frac{D_\nu \overline{E}}{V}. \tag{10}$$

Because of the large number of photons in the enclosure, a classical description of the field suffices to determine D_ν. The states of the system may be represented by a standing wave inside the enclosure, in which case the wavevector has the form

$$\mathbf{k} = \frac{\pi}{V^{1/3}}(n_1, n_2, n_3), \tag{11}$$

where the n_i are integers, and where for simplicity the enclosure is assumed to be a cube with side length $V^{1/3}$. Equation (11) is a statement that an integral number of half wavelengths fit into the box in each of the three directions. For electromagnetic radiation in free space, $k = \omega/c = 2\pi\nu/c$. Equation (11) then implies that the number of states with frequency less than ν corresponds to the number of triplets of integers n_1, n_2, n_3 such that

$$(n_1^2 + n_2^2 + n_3^2)^{1/2} \leq \frac{2V^{1/3}\nu}{c}.$$

The required number of triplets of integers n_1, n_2, n_3 is one eighth of the volume of the sphere with radius $2V^{1/3}\nu/c$ (only positive integers are allowed), i.e. the required number is

$$\frac{1}{8}\frac{4\pi}{3}\left(\frac{2V^{1/3}\nu}{c}\right)^3 = \frac{4\pi}{3}\frac{V\nu^3}{c^3}.$$

This is the number of states with frequency less than ν, i.e. we have established that

$$\int_0^{\nu} D_{\nu'}\,d\nu' = \frac{4\pi}{3}\frac{V\nu^3}{c^3}. \tag{12}$$

Differentiating both sides leads to the required number of photon states per unit frequency:

$$D_\nu = \frac{4\pi V\nu^2}{c^3}. \tag{13}$$

In fact there are two possible polarization states for each photon, so an extra factor of two is needed, giving

$$D_\nu = \frac{8\pi V\nu^2}{c^3}. \tag{14}$$

Putting Equations (9), (10) and (14) together leads to the energy density

$$\rho_\nu(T) = \frac{4\pi}{c}\frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_B T} - 1}. \tag{15}$$

Comparing Equations (15) and (3) confirms that we have obtained the energy density corresponding to the Planck spectrum (2).

3.3 Kirchoff's law revisited

It is worthwhile deriving Kirchoff's law (1), to better understand better its origin. Consider an element of material at temperature T in thermodynamic equilibrium with a blackbody radiation field. In thermodynamic equilibrium there is detailed balance, i.e. the rates of a process and of the inverse process must balance. Specifically, consider the energy emitted per unit time, per unit frequency, per unit area and per unit solid angle in rays about a direction $\boldsymbol{\kappa}$. This quantity depends on the temperature, the direction, and the properties of the material. We will write this quantity $E_\nu(\boldsymbol{\kappa}, T, x)$, where the x denotes the relevant properties of the material. Detailed balance requires that this energy is equal to the energy absorbed (per unit time, per unit frequency, per unit area and per unit solid angle) due to rays propagating in the opposite direction. If a fraction $A_\nu(-\boldsymbol{\kappa}, T, x)$ of the total energy in rays propagating in the direction $-\boldsymbol{\kappa}$ is absorbed, we have

$$E_\nu(\boldsymbol{\kappa}, T, x) = A_\nu(-\boldsymbol{\kappa}, T, x) B_\nu(T), \tag{16}$$

since the ambient intensity is the Planck spectrum $B_\nu(T)$. Equation (16) is a stronger version of Equation (1).

Although Equation (16) is derived in equilibrium, the dependence of the terms $E_\nu(\boldsymbol{\kappa}, T, x)$ and $A_\nu(-\boldsymbol{\kappa}, T, x)$ on temperature and the properties of the material alone mean that it must be true quite generally. To see this, assume Equation (16) describes an element of material on one wall of a cubical blackbody enclosure. We can remove the opposite wall of the enclosure, whilst maintaining the temperature of the other walls. In this situation the element will not be in equilibrium because it will emit more radiation than it absorbs. However, $E_\nu(\boldsymbol{\kappa}, T, x)$ and $A_\nu(-\boldsymbol{\kappa}, T, x)$ are unchanged. Hence the ratio of these quantities is the same, and Equation (16) still holds. It is important to realize that in the latter situation Equation (16) is not a statement about equilibrium: the energy in rays being absorbed by the element is $A_\nu(-\boldsymbol{\kappa}, T, x) I_\nu(-\boldsymbol{\kappa}) < A_\nu(-\boldsymbol{\kappa}, T, x) B_\nu(T)$, where I_ν describes the intensity of the incident radiation.

4. Properties of blackbody radiation

Figure 1 illustrates the Planck spectrum (2) for temperatures $T = 1, 10, 100, ..., 10^8$K (the lowest curve in the figure is the spectrum for $T = 1$K, and the highest curve is for $T = 10^8$K).

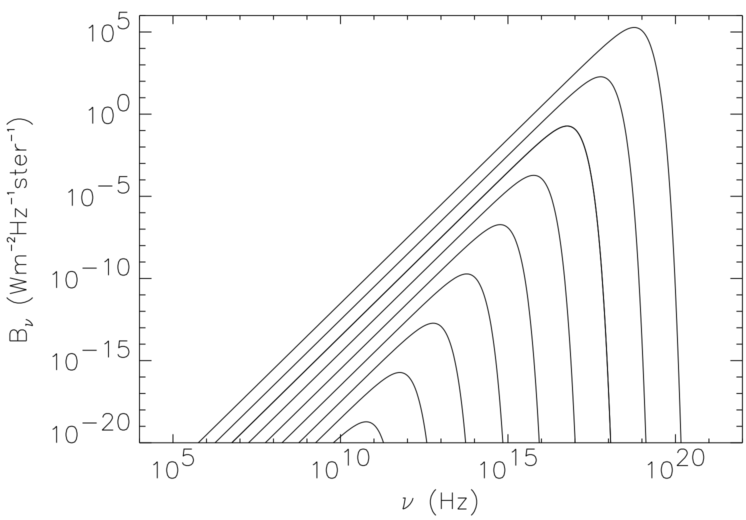

Figure 1: Blackbody spectra for $T = 1, 10, 100, ..., 10^8$K.

Two limiting regimes of behaviour are apparent from the figure, as described in the following two sections.

4.1 The Rayleigh-Jeans law

If $h\nu \ll k_B T$ (i.e. at low frequencies and/or high T) the exponential in the denominator of Equation (2) can be expanded as a series and terms above first order can be dropped. This leads to

$$B_\nu = \frac{2\nu^2}{c^2} k_B T. \tag{17}$$

Equation (17) is the Rayleigh-Jeans law, and corresponds to the power-law behaviour of the spectra at low frequencies in Figure 1.

Interestingly, this is the classical portion of the spectrum. Quantum thermodynamical systems exhibit classical behaviour for large temperature because, according to the Boltzmann law, the probability of occupation of high energy states becomes more likely for large T. At high temperatures there are many accessible energy states, so that there is effectively a continuum of states available, as there is in the classical case.

In § 2 it was mentioned that attempts to derive the blackbody spectrum classically lead to a nonsensical result. In fact they lead to the Rayleigh-Jeans law, which is unphysical as a spectrum over *all* frequencies since it diverges at high frequencies. This behaviour was dramatically called the ultraviolet catastrophe. Looking into a furnace would be lethal based on the X-ray production predicted by Equation (17)!

The classical derivation proceeded as follows. The density of states D_ν in a blackbody enclosure is again given by Equation (14), since that expression was derived classically. The energy associated with each state should be $k_B T$. This follows because for a classical system in thermal equilibrium each degree of freedom of the system has a mean energy $\frac{1}{2}k_B T$. If the cavity is treated as a harmonic oscillator its kinetic energy is on average $\frac{1}{2}k_B T$, and by equipartition the potential energy of the system is also $\frac{1}{2}k_B T$. Hence the classical blackbody energy density is

$$\frac{D_\nu k_B T}{V} = \frac{4\pi}{c}\frac{2\nu^2}{c^2} k_B T,$$

and it follows from Equation (3) that this is the energy density corresponding to the Rayleigh-Jeans law.

4.2 The Wien law

For $h\nu \gg k_B T$ (i.e. high ν and/or low T) the exponential term in the denominator of Equation (2) is much larger than unity, and we have

$$B_\nu = \frac{2h\nu^3}{c^2} e^{-h\nu/k_B T}, \tag{18}$$

which is known as the Wien law. This behaviour corresponds to the steep decline in the spectra at large frequency seen in Figure 1, by which the spectrum avoids the ultraviolet catastrophe.

The Wien law corresponds to the quantum part of the spectrum. However, historically the Wien-law part of the spectrum was known before the Rayleigh-Jeans part, because the experiments are easier at longer wavelengths.

There are two other useful relations for thermal radiation.

4.3 The Stefan-Boltzmann law

Integrating Equation (2) over all frequencies leads to

$$B(T) = \int_0^\infty B_\nu \, d\nu = \frac{2k_B^4 T^4}{c^2 h^3} \int_0^\infty \frac{x^3}{e^x - 1} dx. \tag{19}$$

The integral is found in standard tables and has the value $\pi^4/15$. Hence we have

$$B(T) = \frac{\sigma}{\pi} T^4, \tag{20}$$

where

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} \approx 5.67 \times 10^{-8} \mathrm{Wm}^{-2}\mathrm{K}^{-4} \tag{21}$$

is the Stefan-Boltzmann constant. This is not quite the usual form of the Stefan-Boltzmann law because it is generally formulated in terms of flux F, i.e. the energy per unit time and per unit area crossing a given surface. The quantity $B(T)$ is, according to the discussion at the end of § 2, the energy per unit time, per unit area and per unit solid angle passing a point in space due to rays oriented in a given direction $\boldsymbol{\kappa}$. Suppose a particular bundle of rays is at an angle θ to the surface of interest. Then an element of area dA on the surface presents an area $\cos\theta \, dA$ to the rays, so that the energy per unit time and per unit solid angle crossing the element due to the rays is $B(T) \cos\theta \, dA$. Integrating over solid angle and dividing by the area of the element gives the flux through the surface:

$$F = \int B(T) \cos\theta \, d\Omega = 2\pi B(T) \int_0^\pi \sin\theta \, \cos\theta \, d\theta = \pi B(T). \tag{22}$$

Hence it follows from Equation (20) that the flux associated with a blackbody radiation field is

$$F = \sigma T^4, \tag{23}$$

which is the usual form of the Stefan-Boltzmann law. This describes, in particular, the flux crossing the surface of a blackbody. The steep temperature dependence in this law is apparent in Figure 1: the flux is proportional to the area under each Planck curve, and this clearly increases rapidly with temperature.

4.4 The Wien displacement law

The Wien displacement law gives the value of the frequency, $\nu_{\max}$, at which the Planck spectrum is a maximum. To determine this we need to solve

$$\left. \frac{\partial B_\nu}{\partial \nu} \right|_{\nu=\nu_{\max}} = 0. \tag{24}$$

This leads to the equation $x = 3(1 - e^{-x})$, where $x = h\nu_{\rm max}/(k_BT)$, which has the (approximate) solution $x \approx 2.82$. Hence we have

$$h\nu_{\rm max} \approx 2.82 k_B T, \tag{25}$$

or

$$\nu_{\rm max} \approx 5.88 \times 10^{10} \frac{T}{1\,{\rm K}}\ {\rm Hz}. \tag{26}$$

The linear increase of $\nu_{\rm max}$ with temperature is apparent in Figure 1.

5. Blackbody radiation in astrophysics

Astrophysical sources of radiation that are thermal emitters and that have a state of thermal equilibrium between matter and radiation at the source exhibit blackbody spectra. A variety of other spectra are also observed, corresponding to different emission mechanisms and physical conditions at the source.

5.1 Cosmic Microwave Background radiation

A remarkable example of an astrophysical blackbody spectrum is provided by the Cosmic Microwave Background (CMB), the background radiation in the universe that is believed to be a remnant of the big bang. Figure 2 shows measurements of the CMB from a variety of sources, together with the fit to a blackbody at 2.726 K. The spectrum is very close to the blackbody form. The explanation is that the observed radiation is from the primordial gas of the Universe, and at that time matter and radiation were in thermodynamic equilibrium. Initially the photons were very energetic, but in crossing expanding space they have been redshifted (cooled) to 3 K.

Blackbody radiation is isotropic, but the CMB is anisotropic, for two reasons. First, the motion of the Earth leads to Doppler shifts in the energies of the observed photons, with photons received in the direction of Earth's motion being blueshifted, and photons in the opposite direction redshifted. This effect is easily corrected for. Interestingly, it follows that the CMB provides an absolute reference frame. The second anisotropy is cosmologically significant. The observed radiation exhibits anisotropy at the level of $\Delta T/T \sim 10^{-5}$ which is believed to be intrinsic, and to reflect anisotropies in the early Universe.

5.2 Solar corona

The solar corona is a hot ($T \approx 2 \times 10^6\,{\rm K}$), tenuous ($n \approx 10^9\,{\rm cm}^{-3}$) ionised gas surrounding the Sun, which is visible in scattered light during solar eclipses. Because of its high temperature the solar corona extends out to several solar radii (cf. Equation (5) and our justification of the Boltzmann law). In fact the coronal gas is sufficiently hot that it is not gravitationally bound to the Sun and continuously expands out into interplanetary space, forming the solar wind.

The solar corona emits thermally. If the corona emitted like a blackbody, the peak of its spectrum would be, according to the Wien displacement law, at

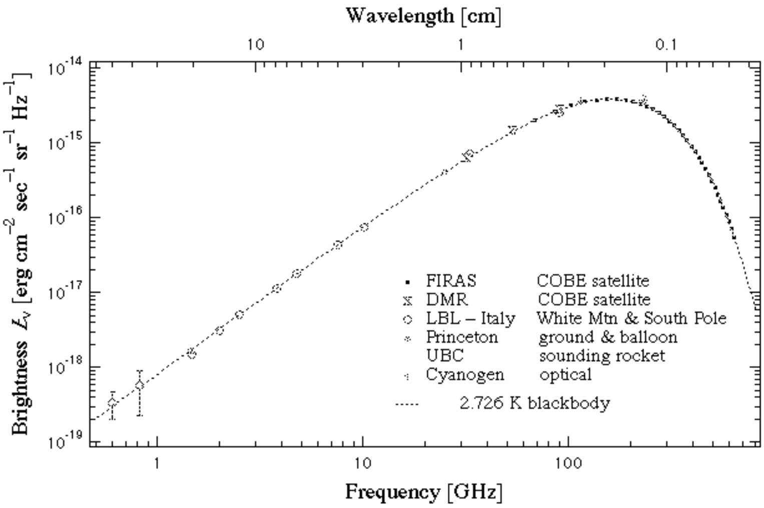

Figure 2: Cosmic microwave background measurements, and the fit to a blackbody spectrum. (From `http://spectrum.lbl.gov/www/cobe/CMB_intensity.gif`.)

an energy $\varepsilon \approx 2.82 k_B T \approx 0.5\,\mathrm{keV}$, which is in the low energy X-ray range. The solar corona does emit most strongly in X-rays. Does the solar corona exhibit a blackbody spectrum?

The answer is no, because the radiation from the solar corona is not in thermal equilibrium with the emitting gas: the coronal gas is too tenuous to absorb much of its own radiation. Despite the non-equilibrium the corona does not cool because it is continually being heated (by a process that is still not well understood).

The non-equilibrium of a hot thin gas may be understood as follows. If the gas was somehow enclosed in a reflecting enclosure the radiation field could be brought into equilibrium with the gas, at the same temperature. The walls would cause emitted radiation to be continually reflected back into the gas, so that the intensity of radiation in the enclosure would rise. The amount of energy absorbed by the gas is proportional to the ambient intensity (see the discussion of Kirchoff's law in § 3.3). Eventually the intensity in the enclosure is high enough for the absorbed energy to match the emitted energy. This happens precisely when the radiation field inside the enclosure is the Planck spectrum, according to Kirchoff's law.

A more practical way for a hot astrophysical gas to be in thermal equilibrium with its radiation field is for the gas to have a sufficiently high density that radiation propagating through the gas is likely to be absorbed. The gas is then termed *optically thick*. Because of the continual absorption and reemission of radiation in such a gas the radiation field is in thermal equilibrium with the material, and a blackbody spectrum results. An example of this situation is the light emerging from the visible surface of the Sun, the photosphere. Beneath the photosphere the solar gas is optically thick and is emitting thermally, so the radiation field is in equilibrium with the material. Hence the spectrum emerging from the photosphere is that of a blackbody, although due to selective absorption by cooler gas higher in the atmosphere, the observed solar spectrum has pieces missing.

To return to the question of the solar corona, in fact the emission from the corona is thermal *bremsstrahlung* ('braking radiation'), produced by acceleration of electrons in the Coulomb fields of ions in the gas. This emission mechanism has a distinctive spectrum that is different from the Planck spectrum, although it also has the same $\sim e^{-h\nu/k_B T}$ behaviour at high frequencies. Although it is strictly beyond the scope of this lecture, I will mention how Kirchoff's law is very useful in this context. Kirchoff's law can be formulated as $j_\nu = \alpha_\nu B_\nu$, where j_ν is the power emitted per unit volume, per unit frequency, and per unit solid angle, and where α_ν is the absorption coefficient, i.e. the fraction of intensity absorbed per unit distance along a ray. The quantity j_ν can be derived, from consideration of the acceleration of electrons passing ions in the gas. Once this is done we get α_ν for free, from Kirchoff's law!

5.3 Measures of temperature

If the radiation from an astrophysical source resembles the blackbody spectrum, then a measure of the temperature of the source can be obtained, by comparison with the Planck spectrum. There are several ways to do this.

The *brightness temperature* is obtained by matching the measured intensity I_ν from a source with that from the Planck law (or one of its approximations), i.e. for a given value of ν the brightness temperature $T_b(\nu)$ is defined by

$$I_\nu = B_\nu[T_b(\nu)], \tag{27}$$

where the right hand side of (27) is the Planck spectrum. At radio frequencies it is common to use the Rayleigh-Jeans law, in which case

$$B_\nu = \frac{2\nu^2}{c^2} k_B T_b \tag{28}$$

and hence

$$T_b = \frac{c^2}{2\nu^2 k_B} I_\nu. \tag{29}$$

If the emitting object is a blackbody, then the brightness temperature will be the temperature of the source. Coherent emission processes (e.g. laser or maser emission, nonlinear emission processes) may produce radiation with a brightness temperature that far exceeds the physical temperature of the source, and this can be used as an indicator of the emission mechamism.

A second approach involves equating the flux at the source with that predicted by the Stefan-Boltzmann law for a blackbody. This leads to the *effective* *temperature*, T_{eff}:

$$F = \sigma T_{\text{eff}}^4. \tag{30}$$

If the emitting object is a blackbody the effective temperature will be the temperature of the source.

An example of effective temperature is provided by the light emitted from the Sun's photosphere. As discussed above, the visible spectrum of the Sun roughly resembles a blackbody, with pieces missing due to absorption in the atmosphere just above the photosphere. Modern spacecraft measurements give the flux of energy from the Sun at the Earth to be $F_{\text{E}} = 1.368\,\text{kW}\,\text{m}^{-2}$. The Earth-Sun distance (an astronomical unit, or AU) is $r_{\text{AU}} = 1.496 \times 10^{11}$m, so the total radiant output of the Sun (the solar luminosity) is $L_{\odot} = 4\pi r_{\text{AU}}^2 F_{\text{E}} = 3.85 \times 10^{26}\,\text{W}$. The radius of the Sun is $R_{\odot} = 6.96 \times 10^8\,\text{m}$, so the flux at the surface of the Sun is $F_{\odot} = L_{\odot}/(4\pi R_{\odot}^2) = 6.32 \times 10^7\,\text{W}\,\text{m}^{-2}$. Equating this with σT_{eff}^4 gives an effective temperature $T_{\text{eff}} = 5778\,\text{K}$. By the Wien displacement law the peak of the visible spectrum of the Sun is at about $5 \times 10^{-7}\,\text{m}$, in the yellow part of the visible spectrum.

Effective temperature (or equivalently colour) is used in astrophysics to categorise stars. For comparison, hot blue-white stars such as Sirius can have an effective temperature around $40,000\,\text{K}$, whereas red giant stars such as Betelguese have $T_{\text{eff}} \approx 3000\,\text{K}$.

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